

Probing the QCD phase diagram with fluctuations[☆]

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Abstract

The relevance of higher order cumulants of conserved charges for the analysis of freeze-out and critical conditions in heavy ion collisions at LHC and RHIC is discussed. Using properties of $O(4)$ scaling functions, the generic structure of these higher cumulants at vanishing baryon chemical potential is discussed. Chiral model calculations are then used to study their properties at non-zero baryon chemical potential. It is argued that the rapid variation of sixth and higher order cumulants at the phase boundary may be used to explore the QCD phase diagram in experiment. Moreover, results for the Polyakov loop susceptibilities in $SU(3)$ lattice gauge theory as well as in $(2+1)$ flavor lattice QCD are discussed. An analysis of the ratios of susceptibilities indicates that the deconfinement transition is reflected in characteristic modifications of these ratios.

Keywords: QCD phase diagram, Phase transitions, Fluctuations

1. Introduction

Strongly interacting matter at high temperature or large net baryon number density is expected to undergo a transition from a phase where chiral symmetry is broken spontaneously to one where chiral symmetry is restored. At vanishing baryon chemical potential ($\mu_B = 0$), this transition is of the crossover type for non-zero quark masses. At $\mu_B > 0$ a second order phase transition point, the so-called chiral critical end point, may exist. A large experimental as well as theoretical effort is invested into the exploration of the QCD phase diagram and the development of appropriate tools and observables that can provide clear-cut signals for the existence of phase transitions and their universal properties. In this context, fluctuations [1], in particular fluctuations of net charges [2, 3], may prove useful.

Critical behavior is connected with long range correlations and enhanced fluctuations, due to the appearance of soft modes in the neighborhood of a phase

[☆]Contribution to “45 Years of Nuclear Theory at Stony Brook: A Tribute to Gerald E. Brown”

transition. Fluctuations of baryon number and electric charge have been shown to reflect the critical behavior of the chiral transition [4]. In the exploration of the QCD phase diagram at non-zero temperature and baryon chemical potential, higher order cumulants of the net baryon number play a particularly important role, since they diverge on the chiral phase transition line in the chiral limit (vanishing light quark masses, $m_q = 0$) and at the conjectured chiral critical end point.

Here I will discuss robust features of cumulants of net baryon number fluctuations that can be extracted from considerations based on $O(4)$ universality, on existing lattice calculations and on model calculations [5].

It has recently been emphasized that also the deconfinement transition can be accurately characterized by exploring fluctuations [6]. In this case the relevant fluctuations are those of the Polyakov loop, the order parameter for the spontaneous breaking of the $\mathcal{Z}(N)$ center symmetry in pure gauge theory. I briefly discuss how this may be used to analyze the deconfinement transition [7].

2. Charge fluctuations and $O(4)$ scaling functions

In studies of critical fluctuations in nucleus-nucleus collisions, the hadron resonance gas (HRG) model, serves as a baseline, since it describes bulk observables [8, 9, 10, 11] and does not exhibit critical behavior. For sufficiently small quark masses, the fluctuations close to the pseudo critical transition line $T_{pc}(\mu_B)$ are expected to reflect the universal properties [12] of the 3-dimensional, $O(4)$ symmetric, spin model [13]. For fluctuations of the net baryon number, the $O(4)$ scaling properties of high order cumulants differ qualitatively from the predictions of the HRG model. Lattice calculations of cumulants of the net baryon number and electric charge, performed in the transition region at vanishing baryon chemical potential and non-zero quark mass, are indeed consistent with the $O(4)$ universality class [4].

Moreover, lattice studies of (2+1) flavor QCD [14] show that for physical light quark masses, the magnetic equation of state is in the $O(N)$ scaling regime. Thus, we can expect that the critical fluctuations at the chiral transition are reflected in suitably chosen observables, which may then be used to pin down the crossover transition in strongly interacting matter. As I will argue, this is indeed the case for higher cumulants of net charges [5].

In nucleus-nucleus collisions, the particle multiplicities are well described in a thermal model using the partition function of a hadron resonance gas [8]. The analysis of data obtained within the HRG model at high beam energies, corresponding to small values of the baryon chemical potential, suggest that the freeze-out curve $T_f(\mu_B)$ is close to the expected QCD phase boundary. Indeed, at $\mu_B = 0$ the chemical freeze-out seems to occur at or very close to the QCD crossover transition [15]. Thus, it is possible that the critical fluctuations of the chiral transition are reflected in fluctuations of, e.g., net charges.

At $\mu_B/T \simeq 0$, the higher cumulants of net charges can be computed within lattice QCD [4, 10]. Eventually such calculations will provide a systematic

theoretical framework for unravelling the relation of the freeze-out conditions at RHIC and LHC energies and the pseudo-critical line in the QCD phase diagram. However, at present lattice calculations provide only limited information on sixth and higher order cumulants. Viable alternatives for discussing qualitative features of the net baryon number fluctuations is offered by $O(4)$ scaling theory and by chiral effective models. In particular, effective models have the advantage that they can be extended to $\mu_B > 0$ with a tractable effort. On the other hand, a clear disadvantage of such models is that they do not account for the potentially important contribution from resonances in the hadronic phase.

Close to the chiral critical point, at $T = T_c, \mu_q = 0, m_q = 0$, the free energy density may be represented in terms of a singular and a regular contribution¹

$$f(T, \mu_q, m_q) = f_s(T, \mu_q, m_q) + f_r(T, \mu_q, m_q). \quad (1)$$

Higher order derivatives of the free energy density with respect to temperature or chemical potential are dominated by the non-analytic (singular) part, f_s . The explicit dependence of the free energy on the chemical potentials of electric charge and strangeness as well as on the strange quark mass is suppressed for simplicity. The singular part of the free energy may be written in the scaling form

$$\frac{f_s(T, \mu_q, h)}{T^4} = A h^{1+1/\delta} f_f(z), \quad z \equiv t/h^{1/\beta\delta}, \quad (2)$$

where β and δ are critical exponents of the 3-dimensional $O(4)$ spin model [13], A is a constant, and the reduced temperature and the symmetry breaking external field are given by [5]

$$\begin{aligned} t &\equiv \frac{1}{t_0} \left(\frac{T - T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T_c} \right)^2 \right), \\ h &\equiv \frac{1}{h_0} \frac{m_q}{T_c}. \end{aligned} \quad (3)$$

Here T_c is the critical temperature in the chiral limit while t_0 and h_0 are non-universal scale parameters [14]. The dependence of the reduced temperature t on the quark chemical potential μ_q , with $\kappa_q \simeq 0.06$, is fixed by charge conjugation symmetry and by requiring that the curvature of the chiral phase boundary at small μ_q/T is reproduced [16].

The scaling properties of the order parameter M are characterized by $M = h^{1/\delta} f_G(z)$, with the scaling function

$$f_G(z) = - \left(1 + \frac{1}{\delta} \right) f_f(z) + \frac{z}{\beta\delta} f'_f(z). \quad (4)$$

The scaling function $f_f(z)$ and its derivatives $f_f^{(n)}(z)$ are known for $n \leq 3$ [17]. These results can be utilized to determine the generic structure of higher order cumulants of the net baryon number fluctuations.

¹From now on I use the chemical potential for quarks μ_q rather than that for baryon number, $\mu_B = 3\mu_q$.

The cumulants of net baryon number are obtained by differentiating the free energy density (1) with respect to $\hat{\mu}_q = \mu_q/T$,

$$\chi_n^B = -\frac{1}{3^n} \frac{\partial^n (f/T^4)}{\partial \hat{\mu}_q^n}. \quad (5)$$

The singular contribution to the cumulants at $\mu_q = 0$ is then given by²

$$(\chi_n^B)_s \sim -h^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z), \quad (6)$$

where I have introduced the specific-heat critical exponent α , using the scaling relation $2 - \alpha = \beta\delta(1 + 1/\delta)$.

Because α is negative for $O(4)$ spin systems in 3 dimensions ($\alpha = -0.2131$ [13]), all cumulants of the net baryon number of order $n \leq 4$, remain finite at the second order transition in the chiral limit ($t = 0, h \rightarrow 0$) for $\mu_q = 0$. Thus, the first divergent cumulant is obtained for $n = 6$. At non-zero μ_q , the odd derivatives survive and the $n = 3$ cumulant is the first one to diverge in the chiral limit. However, for small μ_q/T , the singular part is suppressed by a factor $(\mu_q/T)^3$ relative to that of the sixth order cumulant [5].

Using Eq. (6), one finds the leading singularity in the chiral limit,

$$(\chi_n^B)_s \sim -|t|^{2-\alpha-n/2} f_{\pm}^{(n/2)}, \quad (7)$$

where

$$f_{\pm}^{(n)} = \lim_{z \rightarrow \pm\infty} |z|^{-(2-\alpha-n)} f_f^{(n)}(z). \quad (8)$$

Hence, the singular part of χ_4^B , which is proportional to $f_f^{(2)}$, has the same structure as that of the specific heat; it is proportional to the second derivative of the free energy with respect to temperature. Universality arguments imply [5] that in the chiral limit, the cumulants $\chi_n^B(t)$ of order $n \geq 4$ are all positive for $t < 0$ and alternate in sign for $t > 0$. Consequently, at non-zero quark mass, $h > 0$, one expects χ_6^B to change sign in the transition region and χ_8^B to do so twice, etc.. For a given $h > 0$, this is reflected in the z -dependence of the scaling functions $f_f^{(n)}(z)$ of the 3-dimensional $O(4)$ model [17], as shown in Fig. 1 for different values of the symmetry breaking parameter h . Thus, the generic structure of the n^{th} order cumulant is determined by the corresponding $O(4)$ scaling function, in the chiral limit as well as for non-zero values of the quark mass.

In the chiral limit, the non-analytic contribution to χ_4^B vanishes at the chiral transition temperature, $t = 0$. Consequently, in the transition region, the regular terms dominate in χ_4^B . Nonetheless, the non-analytic term in χ_4^B varies rapidly with temperature, leading to a pronounced maximum in the transition region, observed in lattice as well as in model calculations.

²For $\mu_q = 0$, the odd cumulants vanish, owing to the quadratic dependence on μ_q in (3).

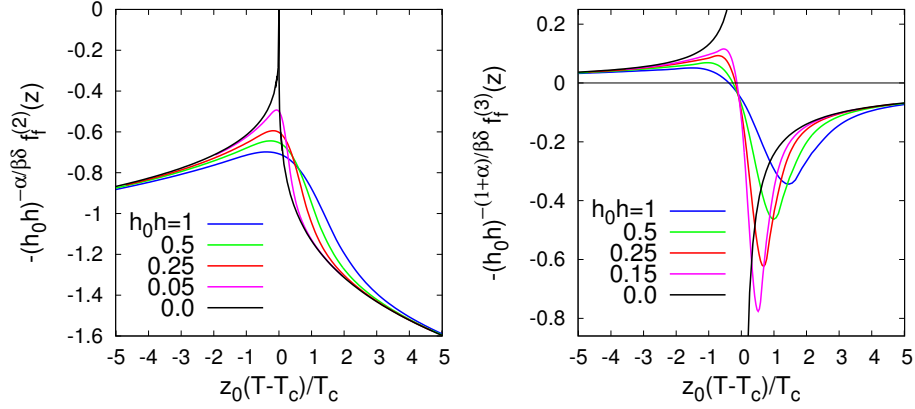


Figure 1: The non-analytic contributions to χ_4^B (left) and χ_6^B (right) arising from second and third derivatives of the singular part of the free energy. Here h_0 and z_0 are non-universal scale parameters [5].

The temperature at which χ_6^B changes sign is non-universal since it depends on the magnitude of the regular terms. However, in the scaling regime the location of the extrema and the corresponding amplitudes follow universal scaling laws, as discussed in [5, 13, 17]. Thus, Eq. (6) implies that the divergent cumulants scale with the correlation length $\xi \sim h^{-\nu/\beta\delta}$ as $\chi_n^B \sim \xi^{-(2-\alpha-n/2)/\nu}$. With the $O(4)$ critical exponents [13], $\alpha = -0.2131$ and $\nu = 0.7377$, one finds $\chi_6^B \sim -\xi^{1.1}$ and $\chi_8^B \sim -\xi^{2.4}$ at $t = 0$. Thus, as expected, the strength of the singularity grows with the order of the cumulant.

We now focus on the properties of the sixth order cumulant, $\chi_6^B(T)$, or correspondingly on the ratio of cumulants $R_{6,2}^B(T) = \chi_6^B(T)/\chi_2^B(T)$. Based on the characteristics of the scaling function, shown in Fig. 1, it is clear that for a sufficiently small, but non-zero, quark mass, $\chi_6^B(T)$ has a maximum in the hadronic phase, close to the transition region and then drops rapidly. Furthermore, just above the critical temperature in the chiral limit, $\chi_6^B(T)$ is negative and exhibits a sharp minimum. Lattice calculations of $\chi_6^B(T)$ [12, 18] indicate that in QCD with physical quark masses, these basic features, which stem from the singular part of χ_6^B , persist. In particular, for the discussion below, it is important that $\chi_6^B(T) < 0$ in the vicinity of the pseudo-critical temperature for chiral symmetry restoration.

The rapid temperature dependence makes the sixth order cumulant a potential probe of the chiral crossover transition in heavy ion collisions. Indeed, if the freeze-out occurs in a temperature range close to the chiral crossover temperature, as indicated by the HRG analysis of particle multiplicities [15], one would expect the data to show a negative sixth order cumulant, in striking contrast to the HRG result, $R_{6,2}^B = 1$. In Fig. 2, $R_{6,2}^B$ is shown in a schematic plot, where both the critical temperature in the chiral limit (T_c) and the crossover temper-

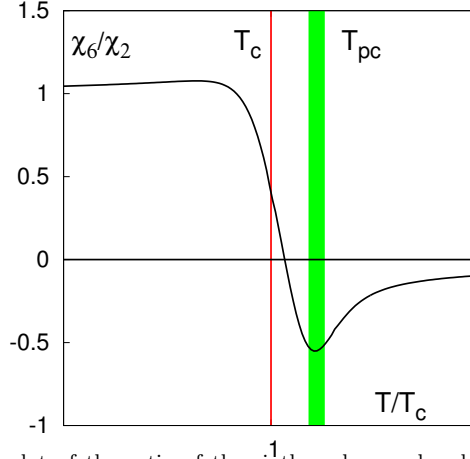


Figure 2: Schematic plot of the ratio of the sixth and second order cumulants of the net baryon number, where T_c is the critical temperature in the chiral limit, while the green band indicates the crossover transition in QCD with physical light quark masses. The pseudo-critical temperature T_{pc} corresponds to a peak in the chiral susceptibility.

ature for chiral symmetry restoration for physical light quark masses (T_{pc}) are indicated. According to scaling arguments, $T_{pc} > T_c$, as indicated in the figure.

Higher order cumulants, e.g. χ_8^B , also show a characteristic behavior near the chiral transition, owing to the singular part of the free energy [5]. As noted above, the strength of the singularity, and thus the signature of critical fluctuations, grows with the order of the cumulant. Therefore, one may naively imagine that the prospects for unraveling the QCD chiral transition could be improved by considering higher cumulants. However, this advantage is counterbalanced by the problems, which arise because with increasing order the cumulants are more and more sensitive to the tail of the probability distribution [19].

3. Fluctuations at $\mu_B/T > 0$

In order to explore whether the rapid temperature dependence of χ_6^B can indeed be utilized to track the phase boundary, one must extend the discussion to non-vanishing baryon chemical potential. This is done in a chiral model, the Polyakov loop extended quark meson model (PQM), within the framework of the functional renormalization group [5, 20]. The chiral transition of the PQM model belongs to the universality class of the 3-dimensional $O(4)$ spin system, while deconfinement is modeled by a $\mathcal{Z}(3)$ symmetric effective potential for the Polyakov loop. Thus, in the light quark mass limit, this model yields the universal scaling functions, discussed in the previous section. Moreover, within this approach the higher cumulants can be computed as functions of μ_q and T . Details on the calculation can be found in [5, 20].

The resulting ratio $R_{6,2}^B$ of the 6th and 2nd order cumulants is shown for $\mu_q/T = 0$ and $\mu_q/T > 0$ in the left panel of Fig. 3. The ratio approaches the hadron resonance gas result at low temperatures, and reproduces the expected $O(4)$ scaling properties in the transition region. In particular, a pronounced

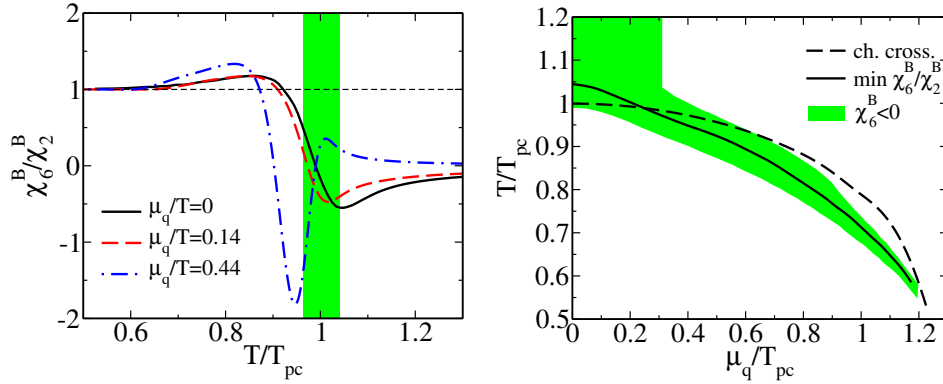


Figure 3: On the left, the temperature dependence of the ratio χ_6^B/χ_2^B for various μ_q/T is shown, while on the right, the location of the chiral crossover line [dashed] and the minimum in χ_6^B [solid line] are displayed. The band shows the range where χ_6^B is negative.

minimum, with $R_{6,2}^B < 0$, is obtained in the vicinity of the chiral crossover temperature. Although the exact location of the minimum and its depth are model dependent, the qualitative structure of the ratio is robust. With increasing μ_q/T , the singular structure becomes stronger and the minimum is shifted to smaller temperatures. This dependence on μ_q/T can be understood in terms of a Taylor expansion of $R_{6,2}^B$ about $\mu_q/T = 0$, where the leading correction in μ_q/T is due to the (more singular) eighth order cumulant [5].

The line of minima of $R_{6,2}^B$ in the μ_q - T plane and the region where the ratio is negative are shown in the right panel of Fig. 3, together with the pseudo critical temperature of the chiral crossover transition obtained with a physical pion mass. At non-zero baryon chemical potential, the temperature interval where χ_6^B (and $R_{6,2}^B(\mu_q/T)$) is negative, shrinks and closely follows the crossover transition line. The region where the sixth order cumulant χ_6^B is negative, extends into the symmetry broken phase. Thus, if freeze-out occurs close to the chiral crossover temperature, the sixth order cumulant of the net baryon number fluctuations will be negative at LHC energies as well as for high RHIC beam energies.

The basic features discussed here for net baryon number fluctuations also carry over to electric charge fluctuations, as indicated by lattice and model calculations [21, 22, 23]. Thus, the corresponding ratio of electric charge cumulants also probes the conditions at freeze-out and their relation to the critical behavior in strongly interacting matter. Moreover, the higher cumulants of electric charge are more directly accessible in experiment than those of the net baryon number.

It should be stressed that there are several uncertainties that may affect the data that are not accounted for in the results presented here. These include acceptance corrections [24, 25], finite volume effects [26], volume fluctuations [27] as well as possible non-equilibrium effects.

Preliminary data by the STAR collaboration [28] show a clear suppression of

the ratio of the sixth order to second order net-proton number cumulant relative to the HRG result. The suppression depends strongly on the beam energy, with the strongest suppression at the lower energies, while the dependence on the centrality of the collision is weak. At low beam energies, the STAR collaboration finds that also the ratio χ_4/χ_2 is suppressed relative to the hadron resonance gas [29]. Such a suppression is expected, since at non-zero μ_q , χ_4 picks up a contribution from χ_6 , which is negative close to the phase boundary [30]. In view of the ideas presented above, the STAR data are quite intriguing. However, a quantitative interpretation of the energy and centrality dependence of the cumulants is not yet available.

4. Fluctuations at the deconfinement transition

The deconfinement transition of QCD is connected with spontaneous breaking of the global $\mathcal{Z}(3)$ center symmetry of color $SU(3)$. The symmetry is exact in the limit of infinitely heavy quarks and is explicitly broken by dynamical quark degrees of freedom. In an $SU(3)$ Yang-Mills theory, the deconfinement transition is first order, while for physical light quark masses it is of the crossover type.

The Polyakov loop is an order parameter of confinement, which is linked with the free energy of a static quark immersed in a hot gluonic medium [31, 32]. At low temperatures, the thermal expectation value of the Polyakov loop $\langle |L| \rangle$ vanishes, indicating color confinement, while at high temperatures $\langle |L| \rangle \neq 0$, corresponding to a finite energy of a static quark, and consequently deconfinement of color and the spontaneous breaking of the $\mathcal{Z}(3)$ symmetry.

In the crossover regime, the order parameter varies smoothly, and does not provide an accurate signature for the transition temperature. Moreover, in Yang-Mills theory on a lattice the Polyakov loop exhibits finite size effects, which smoothens the first order transition, and thus complicates the determination of the transition temperature. It has been shown that the Polyakov loop susceptibilities, in particular their ratios, offer a useful probe of deconfinement, also in full QCD with dynamical light quarks [6, 7].

For the color gauge groups $SU(N_c \geq 3)$, the Polyakov loop is complex-valued. Consequently, the fluctuations of the Polyakov loop are represented by susceptibilities along a longitudinal and a transverse direction³. In lattice calculations, the absolute value of the Polyakov loop and the corresponding susceptibility are commonly used.

³In the real sector of the Polyakov loop, longitudinal and transverse components correspond to the real and imaginary direction respectively. Although the thermal average of the imaginary part of the Polyakov loop vanishes in the real sector, the fluctuations in the imaginary direction do not.

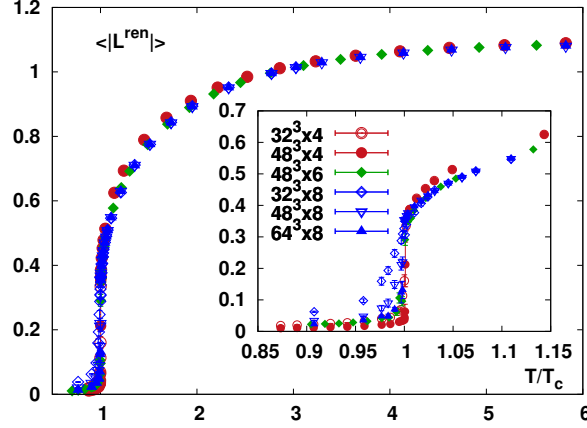


Figure 4: The temperature dependence of the modulus of the renormalized Polyakov loop in SU(3) gauge theory, on various sized lattices. In the insert, finite size effects are clearly visible.

5. Polyakov loop susceptibilities on the lattice

The temperature dependence of the Polyakov loop susceptibilities was computed [6, 7] within SU(3) lattice gauge theory, using the Symanzik improved gauge action on $N_\sigma^3 \times N_\tau$ lattices for different values of the temporal lattice sizes $N_\tau = (4, 6, 8)$ and for spatial extensions N_σ varying from 16 to 64.

On a $N_\sigma^3 \times N_\tau$ lattice, the Polyakov loop is defined as the trace of the product over temporal gauge links,

$$L^{\text{bare}} = \frac{1}{N_c N_\sigma^3} \sum_{\vec{x}} \text{Tr} \prod_{\tau=1}^{N_\tau} U_{(\vec{x}, \tau), 4} \quad (9)$$

while the renormalized Polyakov loop [32] is given by

$$L^{\text{ren}} = (Z(g^2))^{N_\tau} L^{\text{bare}}. \quad (10)$$

The ensemble average of the modulus of the Polyakov loop, $\langle |L^{\text{ren}}| \rangle$, is shown as a function of temperature in Fig. 4 for different sized lattices. This quantity is well defined in the continuum and thermodynamic limits. While no volume effects are visible in the deconfined phase, the results at fixed N_τ in the confined phase, show the expected volume dependence, $\sim 1/\sqrt{V}$.

Using the renormalized Polyakov loop, one can define the renormalized Polyakov loop susceptibility

$$T^3 \chi_A = \frac{N_\sigma^3}{N_\tau^3} (\langle |L^{\text{ren}}|^2 \rangle - \langle |L^{\text{ren}}| \rangle^2). \quad (11)$$

As noted above, in color SU(3) the Polyakov loop operator is complex. Thus, in addition to χ_A , it is useful to consider also the longitudinal and transverse fluctuations of the Polyakov loop [6]

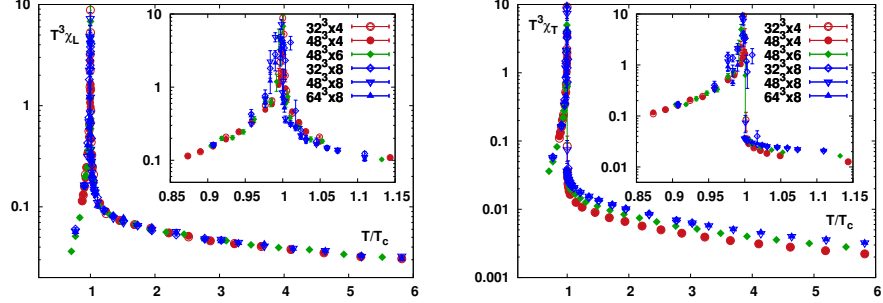


Figure 5: The temperature dependence of the renormalized Polyakov loop susceptibilities from Eqs. (12) and (13) on various lattice sizes, in SU(3) pure gauge theory. The temperature is normalized to the critical value.

$$T^3 \chi_L = \frac{N_\sigma^3}{N_\tau^3} [\langle (L_L^{\text{ren}})^2 \rangle - \langle L_L^{\text{ren}} \rangle^2]. \quad (12)$$

$$T^3 \chi_T = \frac{N_\sigma^3}{N_\tau^3} [\langle (L_T^{\text{ren}})^2 \rangle - \langle L_T^{\text{ren}} \rangle^2], \quad (13)$$

The longitudinal and transverse renormalized Polyakov loop susceptibilities obtained on different lattice sizes are shown in Fig. 5. Near the phase transition, $0.95 < T/T_c < 1.05$, both susceptibilities show a rather strong dependence on the volume, consistent with the first order nature of the phase transition in pure gauge theory. Outside this region, the longitudinal fluctuations of the Polyakov loop, show only a minimal dependence on N_τ and N_σ in both phases. This is the case also for the susceptibility of the modulus, χ_A [6]. The transverse susceptibility χ_T , however, exhibits a residual N_τ dependence in the deconfined phase.

The ambiguities of the renormalization scheme can, to a large extent, be avoided by considering ratios of Polyakov loop susceptibilities [6]. In Fig. 6 the ratios $R_A = \chi_A/\chi_L$ and $R_T = \chi_T/\chi_L$ of susceptibilities obtained in pure SU(3) gauge theory are shown. Both ratios are volume independent and exhibit a strong discontinuity at the deconfinement phase transition. A clear-cut interpretation of the almost temperature independent values of R_A and R_T in the confined phase is obtained using general considerations and the $\mathcal{Z}(3)$ symmetry [6]. Thus, e.g., for temperatures well below T_c the ratio $R_A \simeq 2 - \pi/2$ indicates that the dominant contribution to the susceptibilities is due to Gaussian fluctuations.

The properties of R_T above T_c , shown in the right panel of Fig. 6, indicate that in the SU(3) pure gauge theory, the fluctuations of the longitudinal part of the Polyakov loop are much stronger than those of the transverse part. This is consistent with the shape of the effective Polyakov loop potential, which is ingrained primarily by the $SU(3)$ Haar measure [33].

Also shown in Fig. 6 are results obtained by the HotQCD collaboration with a (2+1) flavor HISQ action and for almost physical quark masses [34, 35]. For

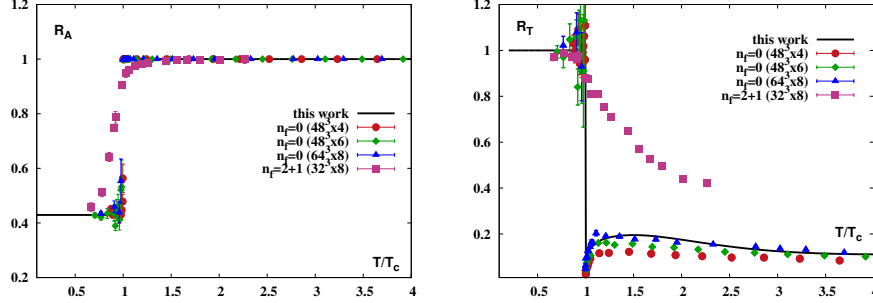


Figure 6: The ratios of Polyakov loop susceptibilities R_A and R_T obtained in lattice gauge theory for a pure gauge system and for (2+1) flavor QCD. The temperature is normalized to its (pseudo) critical value for respective lattice. The lines show the model results, as explained in the text.

$N_f \neq 0$, the temperature in Fig. 6 is normalized to the corresponding pseudo critical temperature. In the presence of dynamical quarks, the Polyakov loop is no longer an order parameter and remains non-zero even in the low temperature phase. Consequently, the ratios of the Polyakov loop susceptibilities are modified due to explicit breaking of the $Z(3)$ symmetry. One therefore expects the smoothening of these ratios across the pseudo critical temperature. Indeed Fig. 6 shows that, in the presence of dynamical quarks, both ratios vary continuously with temperature. The ratio R_A interpolates between the two limiting values set by pure gauge theory, while the width of the crossover region varies with the number of flavors and the quark masses.

In the deconfined phase, the ratio R_T is strongly influenced by dynamical quarks. In addition to the stronger smoothening effect observed, this ratio deviates substantially from the pure gauge result at high T . However, also for small quark masses, the slopes of R_T and R_A change in a narrow temperature range close to the transition point. A quantitative investigation of the effect of quarks on the Polyakov loop susceptibilities and their ratios at the deconfinement transition require systematic studies of the dependence on system size and quark masses.

The lattice results have been used to construct an effective potential for the Polyakov loop, which is consistent with the mean value and the fluctuations of the Polyakov loop as well as with the bulk thermodynamics of pure $SU(3)$ gauge theory [7]. The lines shown in Fig. 6 show the model results for R_A and R_T .

6. Concluding remarks

I have discussed two aspects of fluctuations at the QCD phase boundary. On the one hand, lattice results and model calculations indicate that chiral critical fluctuations are reflected in the higher cumulants of net charges. First data on nucleus-nucleus collisions at RHIC show a suppression of net proton cumulants,

in qualitative agreement with theoretical expectations. However, further studies are needed in order to obtain a quantitative understanding of these phenomena. The deconfinement transition, on the other hand, is associated with fluctuations of the Polyakov loop. I discussed how the Polyakov loop susceptibilities and, in particular, ratios thereof provide a useful probe of the deconfinement transition in lattice QCD. Also here further work is needed to obtain a quantitative probe of deconfinement.

In the mid-70s, when I was a beginning graduate student in Finland, the two potential supervisors were abroad on sabbatical, one of them in Stony Brook. Gerry then agreed to step in as my advisor. For some time I visited him regularly in Copenhagen. However, before long Gerry found that the arrangement was inefficient and suggested that I come to Stony Brook for my PhD. I am deeply grateful to him for his guidance, for many inspiring discussions on physics and on other topics and for his continuous support and friendly advice over many years. I dedicate this paper to his memory.

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